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Effect of Heat Source on Free Convection Fluid Flow in a Vertical Channel with Chemical Reaction

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Abstract

Effect of heat source on free convective fluid flow in a vertical porous channel with chemical reaction has been studied. The problem is solved analytically for approximate solutions for the velocity, temperature and concentration fields. Also, the Skin friction, rate of heat and mass transfer coefficients were derived and shown in tabular form. Effects of various parameters on the flow quantities are presented graphically and discussed. The result shows that there is an excellent agreement with related literatures.

Keywords: Chemical reaction; vertical channel; heat transfer; mass transfer; free convection.

Nomenclature

B_0	External magnetic field	C	Dimensionless concentration
C'	Dimensional concentration of the fluid	C'_w	Constant concentration at the plate
Gr	Thermal Grashof number	g	Acceleration due to gravity
M	Magnetic parameter	N	Mass Grashof number
$\Pr$	Prandtl number	Q	Dimensional heat generation term
R	Chemical reaction parameter	S	Dimensionless heat sink parameter
Sc	Schmidt number	t	Dimensionless time
t_0	Characteristic time	t'	Dimensional time
U	Dimensionless velocity of the fluid	U'	Dimensional velocity of the fluid
y	Dimensionless co-ordinate perpendicular to the plate		
y'	Dimensional co-ordinate to the plate		
τ_0	Skin friction		
Nu_0	Nusselt number		
Sh_0	Sherwood number		

Greek Alphabets

β Volumetric coefficient of thermal expansion
ν Kinematic viscosity
ρ Density of the fluid
θ Fluid Temperature
θ_w Constant temperature at the plate
θ' Dimensional temperature of the fluid
δ Suction

1.0 INTRODUCTION

Free convective flow in presence of heat source has been a subject of interest of many researchers because of its possible applications to Geophysical sciences, Astrophysical sciences, and cosmic studies. Such flows arise either due to unsteady motion of the boundary or boundary temperature.

Choudhary and Das (2014) discussed visco-elastic Magnetohydrodynamics (MHD) free convective flow through porous media in presence of radiation and chemical reaction with heat and mass transfer. Manglesh and Gorla (2013) examined MHD free convective flow through porous medium in the presence of Hall current, radiation and thermal diffusion. Jha (2003) studied transient free convection flow in a vertical channel with heat sink. Mishra *et al.* (2013) analysed free convective flow of viscoelastic fluid in a vertical channel with dufour effect.

The study of heat and mass transfer problems with chemical reaction is of great practical importance to engineers and scientists because of their almost universal occurrence in many branches of science and engineering. A few representative fields of interest in which combined heat and mass transfer along with chemical reaction play an important role are chemical process industries such a food processing and Polymer production. Mahapatra *et al.* (2010) studied effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface. Muthucumaraswamy (2002) illustrated effects of chemical reaction on a moving isothermal vertical surface with suction. Al-Azab and Al-Odat (2007) examined an influence of chemical reaction on transient MHD free convection flow over a moving vertical porous plate. Reddy *et al.* (2015) investigated free convection heat and mass transfer flow of chemically reactive and radiation absorption fluid in an aligned magnetic field. Mandal *et al.* (2014) presented transient free convection in a vertical channel with variable temperature and mass diffusion. Rao *et al.* (2014) studied mixed convective heat and mass transfer flow of a viscous fluid in a vertical channel with thermal radiation and soret effect. Sarma and Govarhan (2016) studied thermo-Diffusion and Diffusion-thermo effects on free convection heat and mass transfer from a vertical surface in a porous medium with viscous dissipation in the presence of radiation.

The aim of the present study is to examine the effects of heat source on free convection fluid flow in a vertical porous channel in the presence of chemical reaction. The governing equations in dimensionless form are solved analytically with a view to obtain solutions for the velocity, temperature and concentration of the fluid. The results were presented with the help of graphs and tables. The effect of parameters controlling the fluid flow is discussed.

1.1 Formulation of the Research Problem

The research problem considers chemical reaction effect on natural convective flow between fixed vertical plates with suction and injection. Figure 1 shows the physical configuration and one of the plate is placed at y=0 and the other at distance y=h

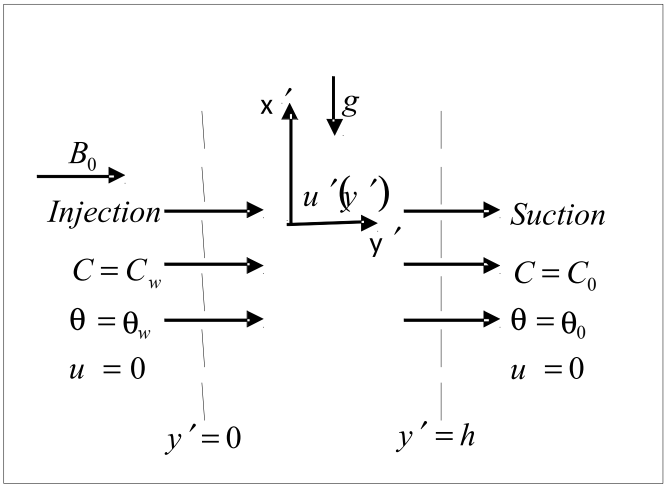

Figure 1. Schematic diagram of the research problem.

The equations governing the flow under the usual boundary layer and Rosseneques approximations are:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2 u'}{\rho} + g\beta(T' - T_0') + g\beta_1(C' - C_0') \tag{2}$$

$$\frac{\partial \theta'}{\partial t'} + v'\frac{\partial \theta'}{\partial y'} = \frac{k}{\rho Cp}\frac{\partial^2 \theta'}{\partial y'^2} + \frac{Q}{\rho Cp}\left(T' - T_0'\right) \tag{3}$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - R\left(C' - C_0'\right) \tag{4}$$

The boundary conditions are:

$$\left.\begin{aligned} &t \le 0, u' = 0, T' = 0 \;\; for \;\; 0 \le y \le h \\ &t > 0, u' = 0, \theta' = T_w' + \varepsilon\left(T_w' - T_0'\right)e^{iwt}, C' = C_w' + \varepsilon\left(C_w' - C_0'\right)e^{iwt} \; at \; y' = 0 \\ &\quad u' = 0, \theta' = \theta_0', C' = C_0' \; at \; y' = h \end{aligned}\right\} \tag{5}$$

In order to write the governing equations and boundary conditions in dimensionless form the following non-dimensional quantities were introduced

$$\left.\begin{aligned} &u = \frac{u'}{u_0}, y = \frac{y'u_0}{H\nu}, t = \frac{t'}{t_0}, \theta = \frac{T' - T_0'}{T_w' - T_0'}, C = \frac{C' - C_0'}{C_w' - C_0'}, \\ &M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, Gr = \frac{g\beta\nu\left(T_w' - T_0'\right)}{u_0^2}, N = \frac{g\beta c\nu\left(C_w' - C_0'\right)}{u_0^2}, \\ &\Pr = \frac{k}{\mu Cp}, Sc = \frac{\nu}{D}, S = \frac{Q\nu}{\rho Cpu_0^2}, \delta = \frac{v_0}{u_0}, R = \frac{R_m \nu}{u_0^2} \end{aligned}\right\} \tag{6}$$

Using equations (1) and (6) on equations (2) to (4) and boundary conditions (5), the dimensionless governing equations become

$$\frac{\partial u}{\partial t}+\delta\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-Mu+Gr\theta+NC \qquad (7)$$

$$\Pr\frac{\partial \theta}{\partial t}+\delta\Pr\frac{\partial \theta}{\partial y}=\frac{\partial^2 \theta}{\partial y^2}+S\Pr\theta \qquad (8)$$

$$Sc\frac{\partial C}{\partial t}+\delta Sc\frac{\partial C}{\partial y}=\frac{\partial^2 C}{\partial y^2}-RScC \qquad (9)$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} t\le 0, u=\theta=C \ \ for\ 0\le y\le 1 \\ t>0, u=0, \theta=1+\varepsilon e^{iwt}, C=1+\varepsilon e^{iwt}\ at\ y=0 \\ \qquad u=0, \theta=0, C=0\, at\ y=1 \end{array}\right\} \qquad (10)$$

To solve equations (7), (8), and (9) subject to the boundary conditions (10), we assume the following for the velocity, temperature and concentration distribution of the field.

$$\left. u=u_0+u_1\varepsilon e^{iwt}, \theta=\theta_0+\theta_1\varepsilon e^{iwt}, C=C_0+C_1\varepsilon e^{iwt}\right\} \qquad (11)$$

where $u_0(y)$, $u_1(y)$, $u_2(y)$, $\theta_0(y)$, $\theta_1(y)$, $\theta_2(y)$, $C_0(y)$, $C_1(y)$ and $C_2(y)$ are to be determined.

Substituting equation (11) in equations (7) to (9), equating harmonic and non harmonic terms, the solving subject to boundary conditions (). We obtain the velocity, temperature and concentration fields respectively as:

$$\left.\begin{array}{l} u(y)=(B_9e^{m_{13}y}+B_{10}e^{-m_{14}y}+B_{11}e^{m_7y}+B_{12}e^{-m_8y}+B_{13}e^{m_1y}+B_{14}e^{-m_2y})+ \\ (B_{15}e^{m_{15}y}+B_{16}e^{-m_{16}y}+B_{17}e^{m_9y}+B_{18}e^{-m_{10}y}+B_{19}e^{m_3y}+B_{20}e^{-m_4y})\varepsilon e^{iwt} \end{array}\right\} \qquad 22$$

$$\theta(y)=B_5e^{m_7y}+B_6e^{-m_8y}+(B_7e^{m_9y}+B_8e^{-m_{10}y})\varepsilon e^{iwt} \qquad 23$$

$$C(y)=B_1e^{m_1y}+B_2e^{-m_2y}+(B_3e^{m_3y}+B_4e^{-m_4y})\varepsilon e^{iwt} \qquad 24$$

Skin Friction

$$\tau_0=\frac{dU}{dy}\Big|_{y=0}=\left.\begin{array}{l}(m_{13}B_9-m_{14}B_{10}-m_7B_{11}-m_8B_{12}-m_1B_{13}-m_2B_{14})+ \\ (m_{15}B_{15}-m_{16}B_{16}-m_9B_{17}-m_{10}B_{18}+m_3B_{19}-m_4B_{20})\varepsilon e^{iwt}\end{array}\right\}$$

Nusselt Number

$$\left. Nu_0=\frac{d\theta}{dy}\right|_{y=0}=\left.\frac{1}{e^{-m_8}-e^{m_7}}\left(m_7e^{-m_8}+m_8e^{m_7}\right)+\frac{\varepsilon}{e^{-m_{10}}-e^{m_9}}\left(m_9e^{-m_{10}}+m_{10}e^{m9}\right)\right\} \qquad 26$$

Sherwood Number

$$\left. Sh=\frac{dC}{dy}\right|_{y=0}=\left.\frac{1}{e^{-m_2}-e^{m_1}}\left(m_1e^{-m_2}+m_2e^{m_3}\right)\right\} \qquad 27$$

3.0 RESULTS AND DISCUSSION

In order to study the physical problem, the velocity, temperature and concentration have been discussed by assigning numerical values to the parameters. The values of Prandtl number (Pr) were chosen to be (Pr = 0.71) for air, (Pr = 7.0) for water and (Pr = 3) for saturated liquid Freon. The values for Schmidtl number were chosen (Sc = 0.22) for hydrogen, (Sc = 0.62) for water vapour, (Sc = 0.8) for ammonia and (Sc= 2.01) for Ethyl benzene. The values of thermal Grashof number (Gr) indicate the state of the channel. Since Gr depends on the channel, it can take positive zero and negative values depending on the temperature of the plates.

Figures 1to 4 showed the behavior of the fluid velocity for various values of parameters were presented. Figure 1 depicts the effect of thermal

Grashof number$\left(Gr \right)$ on fluid velocity; it is observed that the velocity increases with the increase in thermal Grashof number Gr. Figures 1a and 1b reflect that, the volume velocity is higher in water than in the air. The positive values of Gr, indicates the cooling of the channel$\left(Gr > 0 \right)$, it is observed that the velocity is higher. This is due to the fact increase in the thermal Grashof number has the tendency to increase the thermal buoyancy effect, this give rise to an increase in the induced flow. When Gr is negative the reverse effect is observed in case of heating of the channel $\left(Gr < 0 \right)$. In figure 2, the effect of mass Grashof number$\left(N \right)$ on the velocity is shown, it is observed that the velocity increases withincrease in mass Grashof number. It is seen in figures 2a and 2b that the velocity is higher in water $\left(\mathrm{Pr} = 7.00 \right)$ than in the case of air $\left(\mathrm{Pr} = 0.71 \right)$. Figure 3 presents effect of magnetic field on velocity profile, it is clear from this figure that the velocity decreases with the increase in the magnetic field. It is interesting to note that the effect of magnetic field is to decrease the value of velocity profile throughout the boundary layer. The presence of the magnetic field in an electrically conducting fluid introduce Lorentz force, which act against the flow, the velocity flow is greater in the water than in the case of air. As shown in figures 3a and 3b.Figure 4 demonstrates the effect of suction/injection on velocity, velocity increases with the increase of injection for cooling of the channel and decreases for the heating of the channel. It is also clear that suction stabilizes the boundary layer.

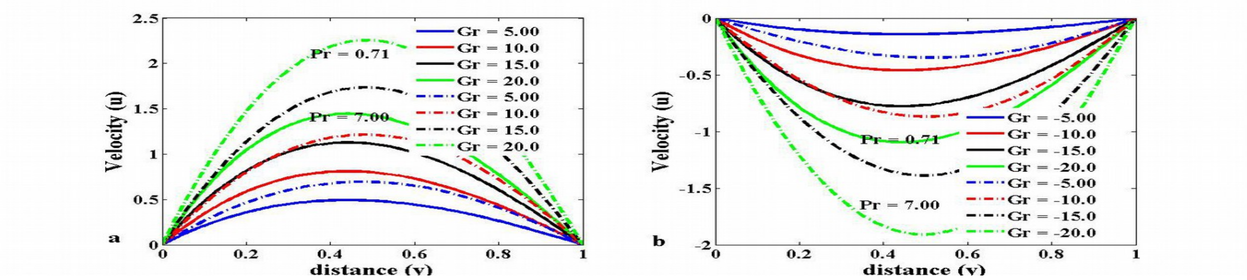

Figure 1. Effect ofthermal Grashof number on the velocity.

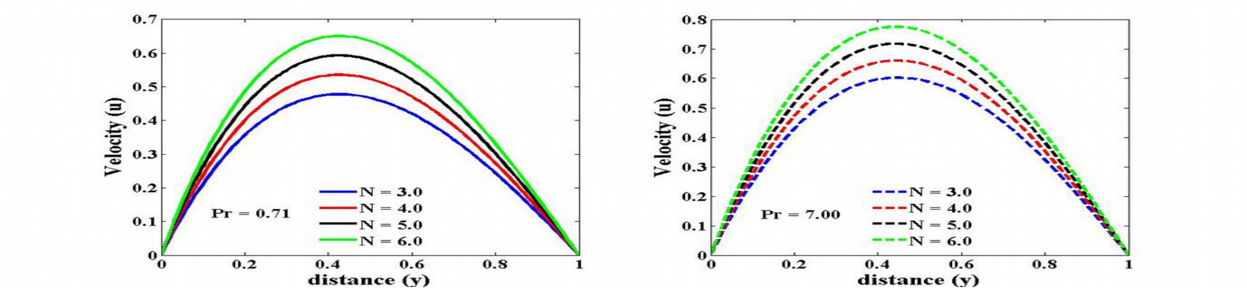

Figure 2: Effect of mass Graaashof number on the velocity.

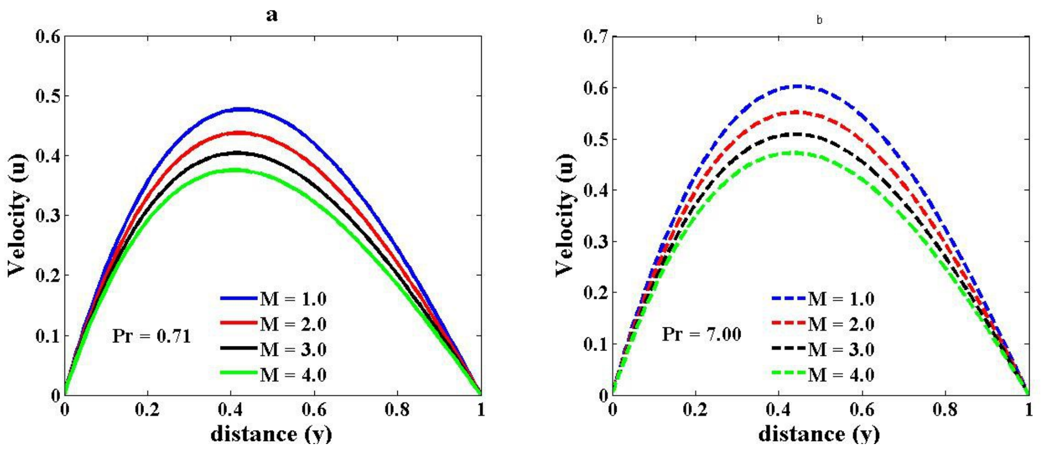

Figure 3. Effect of Magnetic field on the velocity.

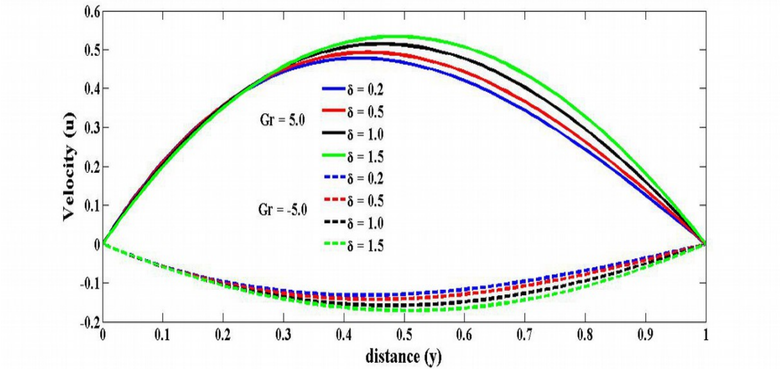

Figure 4: Effect of Suction/Injection on the velocity.

From figure 5, the effects of various parameters on the fluid temperature are illustrated.It is observed that the temperature decreases with the increase as the heat sink parameter. It is seen in figure 6 that, suction/injection is significant on the temperature. The effect of Prandtl number is very important in the temperature field. A fall in temperature occurs due to an increasing values of Prandtl number. This is in agreement with the physical fact that the thermal layer boundary thickness decreases with increase in Pr. Figure 7 demonstrates that the temperature increases with the increase of the suction/injection.

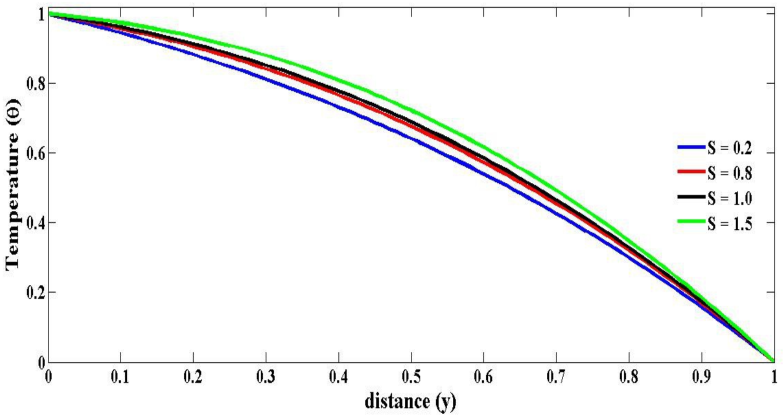

Figure 5. Effect of Heat Sink Parameter on the temperature.

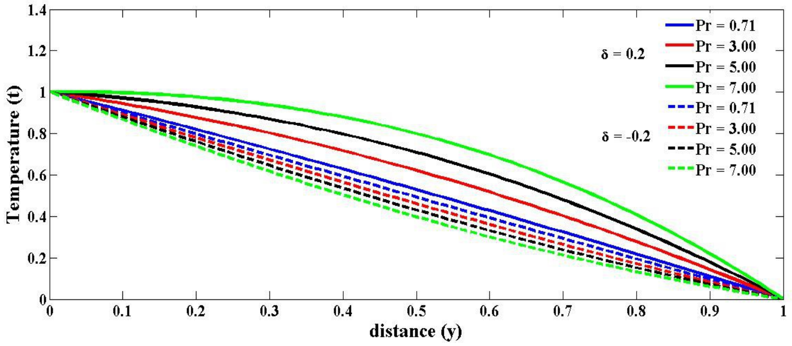

Figure 6. Effect of Prandtl number on the temperature.

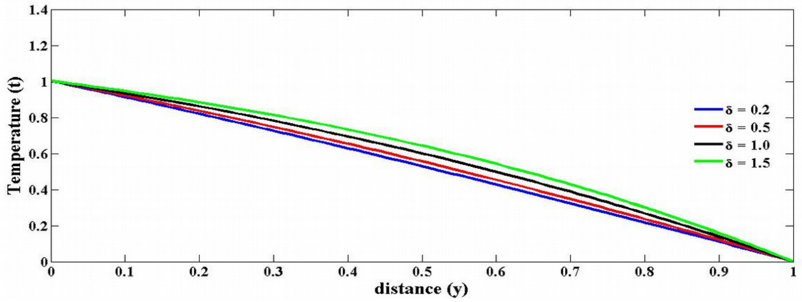

Figure 7. Effect of Suction/Injection on the temperature.

From Figure 8, it is noticed that the increase in Schmidt number decreases the concentration of the fluid this causes the concentration buoyancy to decrease, the reduction in concentration profile are accompanied by reduction in the concentration boundary layer. Figure 9 shows the effect of chemical reaction parameter on the concentration of the fluid. It is observed that the concentration of fluid decreases with the increase in chemical reaction parameter.

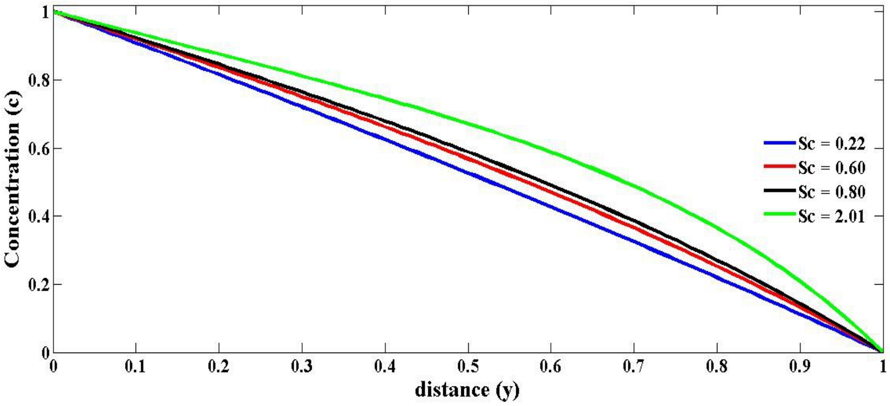

Figure 8. Effect of Schmidt number on the Concentration.

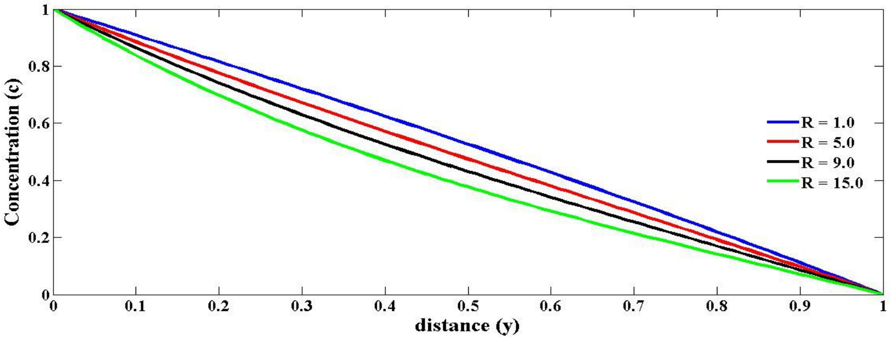

Figure 9. Effect of chemical reaction on the concentration.

Table 1: Skin friction, Nusselt number and Sherwood number.

Pr	Gr	M	R	S	Sc	δ	N	τ_0	Nu_0	Sh_0
0.71	5	1	1	0.2	0.22	0.2	3	2.8972	0.8838	1.0515
3.00	5	1	1	0.2	0.22	0.2	3	2.9092	0.5244	1.0515
7.00	5	1	1	0.2	0.22	0.2	3	3.1910	-0.0233	1.0515
0.71	10	1	1	0.2	0.22	0.2	3	4.2604	0.8838	1.0515
0.71	15	1	1	0.2	0.22	0.2	3	5.6237	0.8838	1.0515
0.71	5	2	1	0.2	0.22	0.2	3	2.4045	0.8838	1.0515
0.71	5	3	1	0.2	0.22	0.2	3	2.2504	0.8838	1.0515
0.71	5	1	2	0.2	0.22	0.2	3	4.3003	0.8838	1.1218
0.71	5	1	3	0.2	0.22	0.2	3	10.3380	0.8838	1.1902
0.71	5	1	1	0.8	0.22	0.2	3	2.8708	0.7346	1.0515
0.71	5	1	1	1	0.22	0.2	3	2.8806	0.6828	1.0515
0.71	5	1	1	0.2	0.60	0.2	3	5.8859	0.8838	1.1347
0.71	5	1	1	0.2	2.01	0.2	3	11.6340	0.8838	1.4056
0.71	5	1	1	0.2	0.22	0.5	3	2.9539	0.7862	1.0193
0.71	5	1	1	0.2	0.22	1.5	3	5.8869	0.5147	0.9170
0.71	5	1	1	0.2	0.22	0.2	4	3.4084	0.8838	1.0515
0.71	5	1	1	0.2	0.22	0.2	5	3.9197	0.8838	1.0515

The comparison of variation of Skin friction $\left(\tau_0\right)$, Nusselt number $\left(Nu_0\right)$ and Sherwood number $\left(Sh_0\right)$ is given in Table 1 above. Increase in Grashof number and mass Grashof number increasesthe skin friction where as there is no effect on Nusselt number and Sherwood number. Increase in the porosity parameter increases the skin friction, Sherwood number without affecting Nusselt number. For increasing Magnetic parameter, the skin friction decreases but Nusselt number and Sherwood number remain unchanged. Increase in Prandtl number is significant on the skin friction but the reverse is the case for Nusselt number no effect is seen in the Sherwood number. Skin friction and Sherwood number increases with the increase in Schmidt number but the Nusselt number remain unchanged.

4.0 CONCLUSION

An analytical solution ofeffect of heat source on free convective fluid flow in a vertical porous channel with chemical reaction has been examined. The governing partial differential equations in dimensionless form are solved by perturbation technique for approximate velocity, temperature and concentration fields. The effects of physical quantities controlling the flow of the fluid are analysed. From the above discussion, we conclude the following:

- The presence of heat source resists the motion of the fluid.
- Lorentz force retards the velocity profile.

- An increase in either thermal Grashof number or mass Grashof number leads to a rise in the fluid velocity. Either Prandtl number or Schmidt number has a retarding influence on the fluid velocity.
- Suction stabilizes the hydrodynamic, thermal as well as concentration boundary layer growth.
- The concentration of the fluid decreases with the increase in chemical reaction parameter.
- The increase in Grashof number causes rise of the Skin friction.
- The increase in Magnetic parameter is significant to the Skin friction.
- The Skin friction and Sherwood number increases with the increase in Schmidt number

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